

Lesson 2-3: Modeling Real – World Data with Matrices | I CAN model data using matrices. | I CAN use a matrix to solve a system of linear equation. | I CAN add, subtract, and multiply matrices.

EQ: How is adding and subtracting matrices different than multiplying matrices?

A **matrix** is an array of numbers and is classified by its dimensions using the number of rows by the number of columns.

5x2 matrix

5 Rows
x
2 Columns

$$\begin{matrix} 1 & \begin{bmatrix} 1 & 4 \\ 6 & 8 \\ 9 & 7 \\ 2 & 0 \\ 4 & 3 \end{bmatrix} \\ 2 & \\ 3 & \\ 4 & \\ 5 & \end{matrix}$$

1

A **square matrix** has the same number of rows and columns.

WHAT IS A SQUARE MATRIX?

$$\begin{bmatrix} 1 & 4 & 0 \\ 8 & 15 & 3 \\ 1 & 9 & 2 \end{bmatrix} \quad \begin{bmatrix} a_{1,1} & a_{1,2} & a_{1,3} \\ a_{2,1} & a_{2,2} & a_{2,3} \\ a_{3,1} & a_{3,2} & a_{3,3} \end{bmatrix}$$

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SQUARE MATRIX 2

Square Matrix

$$\begin{bmatrix} 4 \end{bmatrix} \quad \begin{bmatrix} 4 & -2 \\ -4 & 10 \end{bmatrix} \quad \begin{bmatrix} 2 & 1 & 3 \\ 6 & -3 & 0 \\ 7 & 3 & 5 \end{bmatrix}$$

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Each **element** of a matrix is labeled according to its row and column position.

$$A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1m} \\ a_{21} & a_{22} & \cdots & a_{2m} \\ \vdots & \vdots & & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nm} \end{pmatrix}_{n \times m} = (a_{ij})_{n \times m}$$

m-by-n matrix

$a_{i,j}$ n columns | j changes →

m rows | i changes ↓

$$\begin{bmatrix} a_{1,1} & a_{1,2} & a_{1,3} & \cdots \\ a_{2,1} & a_{2,2} & a_{2,3} & \cdots \\ a_{3,1} & a_{3,2} & a_{3,3} & \cdots \\ \vdots & \vdots & \vdots & \ddots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix}$$

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Equal Matrices:

Two matrices are equal if and only if they have the same dimensions and are identical element by element.

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

Matrix A Matrix B

EX.1 – FINDING DIMENSIONS AND IDENTIFYING ELEMENTS OF A MATRIX

Find the dimensions of each matrix and identify the indicated element.

a. The element a_{12} ,

b. The element a_{31} ,

$$A = \begin{bmatrix} 2 & 3 \\ -1 & 8 \end{bmatrix}$$

$$B = \begin{bmatrix} 3 & -1 & 8 & 3 \\ 2 & 0 & 1 & -4 \\ 5 & 6 & 0 & 9 \end{bmatrix}$$

c. The element a_{43}

$$V = \begin{bmatrix} 12 & 7 & 21 & 31 & 11 \\ 45 & -2 & 14 & 27 & 19 \\ -3 & 15 & 36 & 71 & 26 \\ 4 & -13 & 55 & 34 & 15 \end{bmatrix}$$

Short Summary #1

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EX 2 – USING MATRICES TO SOLVE A SYSTEM OF LINEAR EQUATIONS.

Find the value of x and y for which each matrix equation is true.

a. $[9 \ 13] = [x + 2y \ 4x + 1]$

b.
$$\begin{bmatrix} -12 & 6x \\ 2 & y + 1 \\ 12y & 10 - x \end{bmatrix}$$

c.
$$\begin{bmatrix} x \\ 3y - 1 \end{bmatrix} = \begin{bmatrix} y - 4 \\ 5x - 9 \end{bmatrix}$$

Short Summary #2:

EX.3 – ADDING AND SUBTRACTING MATRICES. (The sum and difference can only be found if the two matrices have the same dimension).

Addition of Matrices:

The sum of two m x n matrices is an m x n matrix in which the elements are the sum of the corresponding elements in the given matrices.

Subtraction of Matrices:

The difference of two m x n matrices A – B is equal to the sum of A + [- B], where – B represents the additive inverse of B.

$$\begin{bmatrix} 3 & 8 \\ 4 & 6 \end{bmatrix} - \begin{bmatrix} 4 & 0 \\ 1 & -9 \end{bmatrix} = \begin{bmatrix} -1 & 8 \\ 3 & 15 \end{bmatrix}$$

Note: A yellow arrow points from the 3 in the top-left of the first matrix to the 4 in the top-left of the second matrix, with the calculation 3-4=-1 written above it.

$$\begin{bmatrix} -4 & 3 \\ 1 & 9 \end{bmatrix} + \begin{bmatrix} 6 & 1 \\ 8 & -2 \end{bmatrix} = \begin{bmatrix} -4+6 & 3+1 \\ 1+8 & 9+(-2) \end{bmatrix} = \begin{bmatrix} 2 & 4 \\ 9 & 7 \end{bmatrix}$$

Note: The final result matrix is enclosed in a blue box and labeled "final answer".

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a. Find $A + B$

$$A = \begin{bmatrix} 2 & -3 \\ 1 & 5 \end{bmatrix} \quad B = \begin{bmatrix} 3 & -4 \\ 8 & -7 \end{bmatrix}$$

b. Find $C + D$

$$C = \begin{bmatrix} -3 & 4 \\ 2 & 11 \end{bmatrix} \quad D = \begin{bmatrix} 0 & -5 \\ 7 & 9 \end{bmatrix}$$

c. Find $A - B$

$$A = \begin{bmatrix} 2 & -3 \\ 1 & 5 \end{bmatrix} \quad B = \begin{bmatrix} 3 & -4 \\ 8 & -7 \end{bmatrix}$$

d. Find $B - A$

Short Summary # 3:

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EX.4 – SCALAR MULTIPLICATION OF MATRICES.

Scalar Product: The product of a scalar k and an $m \times n$ matrix A is an $m \times n$ matrix denoted by kA . Each element of kA equals k times the corresponding element of A .

Matrix Scalar Multiplication

Perform the scalar multiplication.

$$-5 \begin{bmatrix} 4 & 3 \\ -4 & 5 \end{bmatrix} = \begin{bmatrix} -5(4) & -5(3) \\ -5(-4) & -5(5) \end{bmatrix} = \begin{bmatrix} -20 & -15 \\ 20 & -25 \end{bmatrix}$$

If $B = \begin{bmatrix} 3 & -4 \\ -5 & 6 \\ -2 & 0 \end{bmatrix}$, then find $3B$.

3×2

$$3 \begin{bmatrix} 3 & -4 \\ -5 & 6 \\ -2 & 0 \end{bmatrix} = \begin{bmatrix} 3(3) & 3(-4) \\ 3(-5) & 3(6) \\ 3(-2) & 3(0) \end{bmatrix} = \begin{bmatrix} 9 & -12 \\ -15 & 18 \\ -6 & 0 \end{bmatrix}$$

Find the scalar product.

a. If $A = \begin{bmatrix} -4 & 1 & -1 \\ 3 & 7 & 0 \\ -3 & -1 & 8 \end{bmatrix}$, Find $3A$

b. If $A = \begin{bmatrix} 5 & -2 \\ 3 & 8 \\ -1 & 9 \end{bmatrix}$, Find $4A$

Short Summary #4:

EX. 5 – FINDING THE PRODUCT OF TWO MATRICES.

Product of Two Matrices:

The product of an $m \times n$ matrix A and an $n \times r$ matrix B is an $m \times r$ matrix AB . The ij th element in AB is the sum of the products of the corresponding elements in the i th row of A and the j th column of B .

(In order to multiply AB , the number of columns in A has to be the same as the number of rows in B . If they are not, then you cannot multiply because it is not possible and it is said to be undefined).

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Finding the Product of Two Matrices

Find the product. If it is not defined, state the reason.

1. $\begin{bmatrix} 2 & 3 & 4 \\ \textcircled{1} \times 3 \end{bmatrix} \begin{bmatrix} -1 & 4 \\ 0 & 1 \\ 5 & 2 \\ \textcircled{3} \times \textcircled{2} \end{bmatrix}$ To multiply matrices, the number of columns in the first has to be the same as the number of rows in the second. Then multiple row times column

$$\begin{bmatrix} 2 \bullet -1 + 3 \bullet 0 + 4 \bullet 5 & 2 \bullet 4 + 3 \bullet 1 + 4 \bullet 2 \\ -2 + 0 + 20 & 8 + 3 + 8 \end{bmatrix}$$

$$\begin{bmatrix} 18 & 19 \\ \textcircled{1} \times \textcircled{2} \end{bmatrix}$$

Use the given matrices to find each product.

$$A = \begin{bmatrix} 2 & -3 \\ 1 & 5 \end{bmatrix} \quad B = \begin{bmatrix} 3 & -4 \\ 8 & -7 \end{bmatrix}$$

a. Find **AB**

Answer to Part a:

Multiply the first row in A by the first column in B. $2(3) + -3(8) = -18$
 Multiply the first row in A by the second column in B. $2(-4) + -3(-7) = 13$
 Multiply the second row in A by the first column in B. $1(3) + 5(8) = 43$
 Multiply the second row in A by the second column in B. $1(-4) + 5(-7) = -39$

$AB =$

$$\begin{bmatrix} -18 & 13 \\ 43 & -39 \end{bmatrix}$$

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b. Find **BA**

Short Summary #5:

EX.6 – USING A CALCULATOR WITH MATRICES

Calculator Steps:

Entering a matrix in the calculator.

Step 1: Turn **ON** the calculator

Step 2: $2^{\text{nd}}/x^{-1}$ (**MATRX**)

Step 3: Use the right arrow to move to **EDIT**, press **1: [A]**

Step 4: Enter the dimensions of the matrix then enter the elements of the matrix, to move from element to element use the **ENTER** key or the **ARROW** keys. After entering the last element select **ENTER** and then press $2^{\text{nd}}/\text{MODE}$.

Step 5: To perform operations using the matrix press $2^{\text{nd}}/x^{-1}$ then under **NAME** select the number which corresponds to the name of the matrix.

Use the given matrices to perform each operation if possible.

$$A = \begin{bmatrix} 2 & 1 \\ -4 & 7 \end{bmatrix} \quad B = \begin{bmatrix} 6 & -5 \\ 2 & 8 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & -2 \\ 4 & 10 \\ 6 & 0 \end{bmatrix} \quad D = \begin{bmatrix} 3 & -2 & 0 \\ -1 & 4 & 9 \\ 5 & 2 & 12 \end{bmatrix}$$

a. $A + B$

b. $2A$

c. $A - B$

d. AB

e. CD

f. CB

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Short Summary #6:

Independent Practice: HW

Sec. 2-3

P.83 #16-48 even

#16-26

Ex. 2

#28-46

Ex. 3-6

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